SAT Math Cheatsheets

Formula-Rich Notes for Digital SAT Preparation

Algebra — Advanced Math — Problem Solving and Data Analysis — Geometry and Trigonometry

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Use as a revision booklet | **Formula-first format** | **Exam-style notation** | **For practice, tutoring and quick review**

How to Use These Cheatsheets

SAT Math Master Rule

Every question can usually be reduced to one of these moves:

Translate → Set up equation
Solve → Check context

Accuracy Checklist

Units correct? Domain correct? Sign correct?

Did the question ask for x, y, $x+y$, or a coefficient?

Important Symbols

Symbol	Meaning
$\in$	is an element of
$\notin$	is not an element of
$\mathbb{R}$	real numbers
$\mathbb{Z}$	integers
$\varnothing$	empty set
$\leq, \geq$	less/greater than or equal to

Order of Operations

Parentheses → Exponents
Multiply/Divide → Add/Subtract

Cheatsheet 1: Number Sense, Fractions, Ratios and Percents

Fractions

$$\frac{a}{b}+\frac{c}{d}=\frac{ad+bc}{bd}, \qquad \frac{a}{b}-\frac{c}{d}=\frac{ad-bc}{bd}$$
$$\frac{a}{b}\cdot\frac{c}{d}=\frac{ac}{bd}, \qquad \frac{a}{b}\div\frac{c}{d}=\frac{a}{b}\cdot\frac{d}{c}=\frac{ad}{bc}$$
$$\frac{-a}{b}=-\frac{a}{b}=\frac{a}{-b}, \qquad \frac{-a}{-b}=\frac{a}{b}$$

Mixed Numbers and Improper Fractions

$$a\frac{b}{c}=\frac{ac+b}{c}, \qquad \frac{p}{q}=n+\frac{r}{q} \quad \text{where } p=nq+r$$

Decimal and Fraction Conversion

$$0.25=\frac{1}{4}, \quad 0.5=\frac{1}{2}, \quad 0.75=\frac{3}{4}, \quad 0.2=\frac{1}{5}, \quad 0.125=\frac{1}{8}$$
$$0.\overline{3}=\frac{1}{3}, \quad 0.\overline{6}=\frac{2}{3}, \quad 0.\overline{9}=1$$
$$0.\overline{ab}=\frac{10a+b}{99}, \qquad 0.\overline{abc}=\frac{100a+10b+c}{999}$$

Ratios

$$\text{If } a:b=m:n, \quad \frac{a}{b}=\frac{m}{n}$$
$$a=mk, \qquad b=nk, \qquad a+b=(m+n)k$$
$$\text{Part}=\frac{\text{ratio part}}{\text{ratio total}}\cdot\text{whole}$$

Proportions

$$\frac{a}{b}=\frac{c}{d} \iff ad=bc$$
$$\frac{\text{old}}{\text{new}}=\frac{\text{old unit}}{\text{new unit}}, \qquad \frac{x_1}{y_1}=\frac{x_2}{y_2}$$

Percents

$$\text{percent}=\frac{\text{part}}{\text{whole}}\cdot 100$$
$$\text{part}=\frac{p}{100}\cdot\text{whole}, \qquad \text{whole}=\frac{\text{part}}{p/100}$$
$$\text{new}=\text{old}\left(1+\frac{p}{100}\right), \qquad \text{new}=\text{old}\left(1-\frac{p}{100}\right)$$
$$\%\text{ change}=\frac{\text{new}-\text{old}}{\text{old}}\cdot 100$$

Successive Percent Change

$$\text{Final}=\text{Initial}\left(1+\frac{p}{100}\right)\left(1+\frac{q}{100}\right)$$
$$\text{Net change factor}=\prod_{i=1}^{n}\left(1+\frac{p_i}{100}\right)$$

Unit Rate

$$\text{rate}=\frac{\text{amount}}{\text{time}}, \qquad \text{distance}=\text{rate}\cdot\text{time}$$
$$\text{mph}=\frac{\text{miles}}{\text{hours}}$$
$$\text{price per item}=\frac{\text{total price}}{\text{number of items}}$$

Average Speed

$$\text{average speed}=\frac{\text{total distance}}{\text{total time}}$$

For equal distances:

$$\text{average speed}=\frac{2v_1v_2}{v_1+v_2}$$

Work Rate

$$\text{work}=\text{rate}\cdot\text{time}, \qquad \frac{1}{T}=\frac{1}{a}+\frac{1}{b}$$

For three workers:

$$\frac{1}{T}=\frac{1}{a}+\frac{1}{b}+\frac{1}{c}$$

Simple and Compound Interest

$$I=Prt, A=P+I=P(1+rt)$$
$$A=P\left(1+\frac{r}{n}\right)^{nt}, \qquad A=Pe^{rt} \text{ (rare, advanced)}$$

Cheatsheet 2: Algebra Essentials and Linear Equations

Distributive Property

$$a(b+c) = ab + ac, \qquad a(b-c) = ab - ac$$
$$-(a+b) = -a - b, \qquad -(a-b) = -a + b$$

Combining Like Terms

$$ax + bx = (a+b)x, \qquad ax^n + bx^n = (a+b)x^n$$

Only like powers combine: $3x^2 + 5x \neq 8x^2$

Linear Equation

$$ax + b = c \implies x = \frac{c-b}{a} \quad (a \neq 0)$$
$$a(x-h) + k = m \implies x = h + \frac{m-k}{a}$$

No Solution vs Infinitely Many Solutions

Result after simplifying	Meaning
$0 = 5$	no solution
$0 = 0$	infinitely many solutions
$x = 7$	one solution

Solving for a Variable

$$P = 2l + 2w \implies l = \frac{P-2w}{2} = \frac{P}{2} - w$$
$$A = \frac{1}{2}bh \implies h = \frac{2A}{b}$$
$$C = \frac{5}{9}(F-32) \implies F = \frac{9}{5}C + 32$$

Absolute Value

$$|x| = \begin{cases} x, & x \geq 0 \\ -x, & x < 0 \end{cases}$$
$$|x-a| = b \quad (b \geq 0) \implies x = a \pm b$$
$$|x-a| < b \implies a - b < x < a + b$$
$$|x-a| > b \implies x < a - b \text{ or } x > a + b$$

Linear Inequalities

$$ax + b < c \implies ax < c - b$$

If divide by a negative number, reverse the sign:

$$-2x < 8 \implies x > -4$$

Interval Notation

$$(a,b) = a < x < b, \quad [a,b] = a \leq x \leq b$$
$$(-\infty, a) = x < a, \quad [a,\infty) = x \geq a$$

Slope

$$m = \frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1}$$

positive slope: rises, negative slope: falls

$m = 0$: horizontal, undefined: vertical

Line Forms

$$y = mx + b$$
$$y - y_1 = m(x - x_1)$$
$$Ax + By = C$$
$$\frac{x}{a} + \frac{y}{b} = 1 \quad \text{where intercepts are } (a,0), (0,b)$$

Parallel and Perpendicular Lines

Parallel: $m_1 = m_2$

Perpendicular: $m_1 m_2 = -1$

If $m = \frac{a}{b}$, $m_\perp = -\frac{b}{a}$

Intercepts

x-intercept: set $y = 0$

y-intercept: set $x = 0$

$$Ax + By = C: \quad x\text{-int} = \frac{C}{A}, \qquad y\text{-int} = \frac{C}{B}$$

Cheatsheet 3: Systems, Word Problems and Modeling

System of Two Linear Equations

$$\begin{cases} a_1x + b_1y = c_1 \\ a_2x + b_2y = c_2 \end{cases}$$

Elimination idea:

make coefficients opposite, then add equations

Substitution idea:

$$x = \text{expression in } y$$

$$\Rightarrow \text{ substitute into other equation}$$

Systems by Slope

Condition	Number of solutions
$m_1 \neq m_2$	1
$m_1 = m_2,\ b_1 \neq b_2$	0
$m_1 = m_2,\ b_1 = b_2$	∞

Linear Combination Shortcut

If

$$a + b = 10, \qquad a - b = 4$$

then

$$2a = 14 \Rightarrow a = 7, \qquad 2b = 6 \Rightarrow b = 3$$

Mixture Formula

$$\text{pure amount} = \text{concentration} \cdot \text{total amount}$$

$$c_1x + c_2y = c_f(x + y)$$

Weighted Average

$$\bar{x}_w = \frac{w_1x_1 + w_2x_2 + \cdots + w_nx_n}{w_1 + w_2 + \cdots + w_n}$$

If weights are percents:

$$\bar{x}_w = p_1x_1 + p_2x_2 + \cdots + p_nx_n, \qquad \sum p_i = 1$$

Average Formula

$$\text{average} = \frac{\text{sum}}{\text{number}}$$

$$\text{sum} = \text{average} \cdot \text{number}$$

$$\text{new average} = \frac{\text{old sum} + \text{new value}}{\text{old count} + 1}$$

Direct Variation

$$y = kx, \qquad k = \frac{y}{x}$$

$$\frac{y_1}{x_1} = \frac{y_2}{x_2}$$

Inverse Variation

$$y = \frac{k}{x}, \qquad xy = k$$

$$x_1y_1 = x_2y_2$$

Joint Variation

$$z = kxy, \qquad z = k\frac{x}{y}, \qquad z = kx^ay^b$$

Revenue, Cost and Profit

$$\text{Revenue} = \text{price} \cdot \text{quantity}$$

$$\text{Profit} = \text{Revenue} - \text{Cost}$$

$$\text{Break-even: Revenue} = \text{Cost}$$

Distance Problems

$$d = rt, \qquad r = \frac{d}{t}, \qquad t = \frac{d}{r}$$

Same direction:

$$\text{gap closed} = (r_1 - r_2)t$$

Opposite directions:

$$\text{distance apart} = (r_1 + r_2)t$$

Digit Problems

Two-digit number:

$$10a + b$$

Three-digit number:

$$100a + 10b + c$$

Reverse of two-digit number:

$$10b + a$$

Cheatsheet 4: Exponents, Radicals and Scientific Notation

Exponent Laws

$$a^m \cdot a^n = a^{m+n}$$
$$\frac{a^m}{a^n} = a^{m-n} \quad (a \neq 0)$$
$$(a^m)^n = a^{mn}$$
$$(ab)^n = a^n b^n, \qquad \left(\frac{a}{b}\right)^n = \frac{a^n}{b^n}$$

Zero and Negative Exponents

$$a^0 = 1 \quad (a \neq 0)$$
$$a^{-n} = \frac{1}{a^n}, \qquad \frac{1}{a^{-n}} = a^n$$

Fractional Exponents

$$a^{1/n} = \sqrt[n]{a}$$
$$a^{m/n} = \sqrt[n]{a^m} = (\sqrt[n]{a})^m$$
$$a^{-m/n} = \frac{1}{a^{m/n}}$$

Radical Product and Quotient

$$\sqrt{ab} = \sqrt{a}\sqrt{b}$$
$$\sqrt{\frac{a}{b}} = \frac{\sqrt{a}}{\sqrt{b}} \quad (b > 0)$$
$$\sqrt{a^2} = |a|$$

Simplifying Radicals

$$\sqrt{50} = \sqrt{25 \cdot 2} = 5\sqrt{2}$$
$$\sqrt{a^2 b^3} = |a| b\sqrt{b} \quad (b \geq 0)$$

Rationalizing Denominators

$$\frac{1}{\sqrt{a}} = \frac{\sqrt{a}}{a}$$
$$\frac{1}{a + \sqrt{b}} = \frac{a - \sqrt{b}}{a^2 - b}$$
$$(a + \sqrt{b})(a - \sqrt{b}) = a^2 - b$$

Scientific Notation

$$N = a \times 10^n, \qquad 1 \leq |a| < 10$$
$$(3.2 \times 10^5)(4 \times 10^{-2}) = 12.8 \times 10^3 = 1.28 \times 10^4$$

Growth and Decay

$$y = a(1 + r)^t \quad \text{growth}$$
$$y = a(1 - r)^t \quad \text{decay}$$
$$r = \text{growth/decay rate as decimal}$$

Exponential Equation Basics

$$a^x = a^y \implies x = y \quad (a > 0,\ a \neq 1)$$
$$2^{x+1} = 16 = 2^4 \implies x + 1 = 4$$

Geometric Sequences

$$a_n = a_1 r^{n-1}$$
$$\text{common ratio } r = \frac{a_n}{a_{n-1}}$$
$$S_n = a_1 \frac{1 - r^n}{1 - r} \quad (r \neq 1)$$

Arithmetic Sequences

$$a_n = a_1 + (n - 1)d$$
$$\text{common difference } d = a_n - a_{n-1}$$
$$S_n = \frac{n}{2}(a_1 + a_n)$$

Comparing Linear and Exponential

Linear: $y = mx + b \quad \Rightarrow \quad$ constant difference

Exponential: $y = ab^x \quad \Rightarrow \quad$ constant ratio

Cheatsheet 5: Factoring, Polynomials and Rational Expressions

Common Factor

$$ab + ac = a(b + c)$$
$$6x^3 + 9x^2 = 3x^2(2x + 3)$$

Difference of Squares

$$a^2 - b^2 = (a - b)(a + b)$$
$$x^2 - 16 = (x - 4)(x + 4)$$

Perfect Square Trinomials

$$a^2 + 2ab + b^2 = (a + b)^2$$
$$a^2 - 2ab + b^2 = (a - b)^2$$

Quadratic Trinomial

$$x^2 + (m + n)x + mn = (x + m)(x + n)$$
$$ax^2 + bx + c = (px + q)(rx + s)$$

where

$$pr = a, \quad qs = c, \quad ps + qr = b$$

Sum and Difference of Cubes

$$a^3 + b^3 = (a + b)(a^2 - ab + b^2)$$
$$a^3 - b^3 = (a - b)(a^2 + ab + b^2)$$

Polynomial Operations

$$(ax^m)(bx^n) = abx^{m+n}$$
$$\frac{ax^m}{bx^n} = \frac{a}{b}x^{m-n}$$
$$(2x + 3)(x - 5) = 2x^2 - 10x + 3x - 15$$

Rational Expression Restrictions

$$\frac{P(x)}{Q(x)} \text{ is undefined when } Q(x) = 0$$
$$\frac{x^2 - 9}{x - 3} = x + 3, \quad x \neq 3$$

Rational Expression Operations

$$\frac{a}{b} + \frac{c}{d} = \frac{ad + bc}{bd}$$
$$\frac{a}{b} \cdot \frac{c}{d} = \frac{ac}{bd}$$
$$\frac{\frac{a}{b}}{\frac{c}{d}} = \frac{a}{b} \cdot \frac{d}{c}$$

Zeros and Factors

$$P(r) = 0 \iff (x - r) \text{ is a factor of } P(x)$$
$$\text{If } P(x) = a(x - r_1)(x - r_2), \quad \text{zeros are } r_1, r_2$$

Remainder Theorem

Remainder when $P(x)$ is divided by $x - a$ is $P(a)$

End Behavior - Leading Term

For large $|x|$,

$$P(x) = a_n x^n + \cdots \approx a_n x^n$$

n	$a_n > 0$	$a_n < 0$
even	$\uparrow, \uparrow$	$\downarrow, \downarrow$
odd	$\downarrow, \uparrow$	$\uparrow, \downarrow$

Useful Identities

$$(a + b)^2 = a^2 + 2ab + b^2$$
$$(a - b)^2 = a^2 - 2ab + b^2$$
$$(a + b)(a - b) = a^2 - b^2$$
$$(a + b)^3 = a^3 + 3a^2b + 3ab^2 + b^3$$
$$(a - b)^3 = a^3 - 3a^2b + 3ab^2 - b^3$$

Cheatsheet 6: Quadratics and Nonlinear Functions

Standard Form

$$y = ax^2 + bx + c$$

$$\text{y-intercept} = c$$

$$\text{axis of symmetry: } x = -\frac{b}{2a}$$

$$\text{vertex: } \left(-\frac{b}{2a},\ f\left(-\frac{b}{2a}\right)\right)$$

Vertex Form

$$y = a(x-h)^2 + k$$

$$\text{vertex} = (h,k), \qquad \text{axis } x = h$$

$$a > 0 \Rightarrow \text{opens up}, \qquad a < 0 \Rightarrow \text{opens down}$$

Factored Form

$$y = a(x-r_1)(x-r_2)$$

$$\text{x-intercepts } (r_1,0), (r_2,0)$$

$$\text{axis } x = \frac{r_1 + r_2}{2}$$

Quadratic Formula

$$ax^2 + bx + c = 0$$

$$\boxed{x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}}$$

Discriminant

$$\Delta = b^2 - 4ac$$

$\Delta > 0$	two real solutions
$\Delta = 0$	one real repeated solution
$\Delta < 0$	no real solutions

Completing the Square

$$x^2 + bx = \left(x + \frac{b}{2}\right)^2 - \left(\frac{b}{2}\right)^2$$

$$ax^2 + bx + c = a\left(x + \frac{b}{2a}\right)^2 + c - \frac{b^2}{4a}$$

Sum and Product of Roots

If $ax^2 + bx + c = 0$, roots r_1, r_2:

$$r_1 + r_2 = -\frac{b}{a}, \qquad r_1 r_2 = \frac{c}{a}$$

Maximum or Minimum Value

$$y = a(x-h)^2 + k$$

$$\begin{cases} \text{minimum} = k, & a > 0 \\ \text{maximum} = k, & a < 0 \end{cases}$$

Parabola Transformations

$$y = a(x-h)^2 + k$$

$$|a| > 1: \text{ vertical stretch}$$

$$0 < |a| < 1: \text{ vertical compression}$$

$$h > 0: \text{ shift right}, \quad k > 0: \text{ shift up}$$

Function Notation

$$f(x) = 2x + 3 \quad \Rightarrow \quad f(a) = 2a + 3$$

$$f(x+h) = 2(x+h) + 3 = 2x + 2h + 3$$

Composition of Functions

$$(f \circ g)(x) = f(g(x))$$

$$(g \circ f)(x) = g(f(x))$$

Usually

$$f(g(x)) \neq g(f(x))$$

Inverse Function Basics

$$f^{-1}(f(x)) = x, \qquad f(f^{-1}(x)) = x$$

To find inverse:

$$y = f(x) \Rightarrow x = f(y) \Rightarrow y = f^{-1}(x)$$

Domain and Range

$$\text{Domain} = \text{allowed } x\text{-values}$$

$$\text{Range} = \text{possible } y\text{-values}$$

Common restrictions:

$$\text{denominator} \neq 0, \qquad \text{inside even root} \geq 0$$

Piecewise Functions

$$f(x) = \begin{cases} 2x + 1, & x < 3 \\ x^2, & x \geq 3 \end{cases}$$

Use the rule whose condition includes the input.

Cheatsheet 7: Coordinate Geometry and Graphs

Distance Formula

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

Midpoint Formula

$$M = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right)$$

Slope Again

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

$$\text{rise} = y_2 - y_1, \qquad \text{run} = x_2 - x_1$$

Equation of a Circle

$$(x - h)^2 + (y - k)^2 = r^2$$

$$\text{center} = (h, k), \qquad \text{radius} = r$$

Circle from General Form

$$x^2 + y^2 + Dx + Ey + F = 0$$

Complete square:

$$(x + \frac{D}{2})^2 + (y + \frac{E}{2})^2 = \frac{D^2 + E^2}{4} - F$$

$$\text{center} = \left(-\frac{D}{2}, -\frac{E}{2} \right)$$

Area on Coordinate Plane

Rectangle with side lengths:

$$A = (\Delta x)(\Delta y)$$

Triangle using base and height:

$$A = \frac{1}{2} bh$$

Shoelace formula for vertices (x_i, y_i):

$$A = \frac{1}{2} \left| \sum x_i y_{i+1} - \sum y_i x_{i+1} \right|$$

Graph Transformations

$$y = f(x) + k \quad \text{up } k$$

$$y = f(x - h) \quad \text{right } h$$

$$y = -f(x) \quad \text{reflection over x-axis}$$

$$y = f(-x) \quad \text{reflection over y-axis}$$

$$y = af(x) \quad \text{vertical stretch/compression}$$

Linear Function Graph

$$y = mx + b$$

$$\text{slope} = m, \qquad y\text{-intercept} = b$$

$$\Delta y = m \Delta x$$

Exponential Graph

$$y = ab^x$$

$$a = \text{initial value}, \qquad b = \text{growth factor}$$

$$b > 1 : \text{growth}, \quad 0 < b < 1 : \text{decay}$$

Scatterplots and Line of Best Fit

$$\hat{y} = mx + b$$

$$\text{residual} = y - \hat{y}$$

positive residual: actual above prediction

negative residual: actual below prediction

Interpreting Slope in Context

$$\frac{\Delta y}{\Delta x} = \frac{\text{change in output}}{\text{change in input}}$$

Example units:

$$\frac{\text{dollars}}{\text{hour}}, \quad \frac{\text{miles}}{\text{gallon}}, \quad \frac{\text{score points}}{\text{week}}$$

Intercept Meaning in Context

$$y = mx + b$$

$b =$ predicted value when $x = 0$

x-intercept = input where output is 0

Cheatsheet 8: Statistics, Data Analysis and Probability

Mean, Median, Mode, Range

$$\text{mean} = \bar{x} = \frac{x_1 + x_2 + \cdots + x_n}{n}$$

$$\text{median} = \text{middle value after sorting}$$

$$\text{mode} = \text{most frequent value}$$

$$\text{range} = \max - \min$$

Quartiles and IQR

Q_1 = median of lower half, Q_3 = median of upper half

$$\text{IQR} = Q_3 - Q_1$$

Outlier rule:

$$x < Q_1 - 1.5(\text{IQR}) \quad \text{or} \quad x > Q_3 + 1.5(\text{IQR})$$

Standard Deviation Concept

$$\sigma = \sqrt{\frac{\sum (x_i - \mu)^2}{N}}$$

$$s = \sqrt{\frac{\sum (x_i - \bar{x})^2}{n-1}}$$

Larger standard deviation means more spread.

Weighted Average

$$\bar{x} = \frac{\sum w_i x_i}{\sum w_i}$$

Percentile

$$\text{percentile rank} = \frac{\#\text{ values below}}{\#\text{ total values}} \cdot 100$$

Two-Way Table Probability

$$P(A) = \frac{\#A}{\#\text{total}}$$

$$P(A \cap B) = \frac{\#\text{in both } A \text{ and } B}{\#\text{total}}$$

$$P(A \mid B) = \frac{\#\text{in both } A \text{ and } B}{\#B}$$

Probability Rules

$$0 \le P(A) \le 1$$

$$P(A^c) = 1 - P(A)$$

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

If mutually exclusive:

$$P(A \cup B) = P(A) + P(B)$$

Independent Events

$$P(A \cap B) = P(A)P(B)$$

$$P(A \mid B) = P(A)$$

Dependent Events

$$P(A \cap B) = P(A)P(B \mid A)$$

Without replacement changes the denominator.

Counting Principle

$$\text{Total outcomes} = m_1 m_2 m_3 \cdots m_n$$

Permutations and Combinations

$${}_nP_r = \frac{n!}{(n-r)!}, \qquad \binom{n}{r} = \frac{n!}{r!(n-r)!}$$

P : order matters, C : order does not

Expected Value

$$E(X) = \sum x_i P(x_i)$$

Sampling and Margin of Error

$$\text{likely interval} = \text{estimate} \pm \text{margin of error}$$

$$\hat{p} = \frac{x}{n}$$

Correlation

$$-1 \le r \le 1$$

$r \approx 1$: positive, $r \approx -1$: negative, $r \approx 0$: weak

correlation $\neq$ causation

Cheatsheet 9: Geometry - Lines, Angles, Triangles and Polygons

Angle Facts

$$\text{straight line} = 180^\circ, \qquad \text{full turn} = 360^\circ$$

vertical angles are equal

linear pair sums to 180°

complementary: $a + b = 90^\circ$

supplementary: $a + b = 180^\circ$

Parallel Lines

With parallel lines cut by a transversal:

corresponding angles equal

alternate interior angles equal

same-side interior angles sum to 180°

Triangle Angle Sum

$$A + B + C = 180^\circ$$

Exterior angle:

$$\text{exterior} = \text{remote interior}_1 + \text{remote interior}_2$$

Triangle Area

$$A = \frac{1}{2}bh$$

Heron's formula:

$$s = \frac{a+b+c}{2}, \qquad A = \sqrt{s(s-a)(s-b)(s-c)}$$

Pythagorean Theorem

$$a^2 + b^2 = c^2$$

$$c = \sqrt{a^2 + b^2}$$

right triangle only, c = hypotenuse

Special Right Triangles

$$45^\circ - 45^\circ - 90^\circ: \quad x, x, x\sqrt{2}$$

$$30^\circ - 60^\circ - 90^\circ: \quad x, x\sqrt{3}, 2x$$

Similar Triangles

$$\frac{\text{side}_1}{\text{matching side}_1} = \frac{\text{side}_2}{\text{matching side}_2} = k$$

$$\frac{\text{perimeters}}{} = k, \qquad \frac{\text{areas}}{} = k^2$$

Triangle Inequality

$$a + b > c, \qquad a + c > b, \qquad b + c > a$$

For third side x:

$$|a - b| < x < a + b$$

Polygon Interior Angles

$$\text{sum of interior angles} = (n-2)180^\circ$$

$$\text{one interior angle of regular } n\text{-gon} = \frac{(n-2)180^\circ}{n}$$

$$\text{one exterior angle of regular } n\text{-gon} = \frac{360^\circ}{n}$$

Quadrilaterals

parallelogram: opposite sides parallel and equal

rectangle: four right angles

rhombus: four equal sides

square: rectangle + rhombus

Areas of Common Shapes

$$A_{\text{rectangle}} = lw, \qquad A_{\text{square}} = s^2$$

$$A_{\text{parallelogram}} = bh, \qquad A_{\text{triangle}} = \frac{1}{2}bh$$

$$A_{\text{trapezoid}} = \frac{1}{2}(b_1 + b_2)h$$

$$A_{\text{rhombus/kite}} = \frac{1}{2}d_1 d_2$$

Perimeters

$$P_{\text{rectangle}} = 2l + 2w, \qquad P_{\text{square}} = 4s$$

$$P_{\text{triangle}} = a + b + c$$

$$P_{\text{regular } n\text{-gon}} = ns$$

Cheatsheet 10: Circles, Arcs, Sectors and Solids

Circle Basics

$$C = 2\pi r = \pi d$$

$$A = \pi r^2$$

$$d = 2r$$

Arc Length

$$\text{arc length} = \frac{\theta}{360^\circ} \cdot 2\pi r$$

If θ is in radians:

$$\text{arc length} = r\theta$$

Sector Area

$$A_{\text{sector}} = \frac{\theta}{360^\circ} \cdot \pi r^2$$

If θ is in radians:

$$A_{\text{sector}} = \frac{1}{2} r^2 \theta$$

Central and Inscribed Angles

$$\text{central angle} = \text{intercepted arc}$$

$$\text{inscribed angle} = \frac{1}{2}(\text{intercepted arc})$$

Tangent Radius Rule

radius to tangent point $\perp$ tangent line

Chord and Diameter Facts

diameter perpendicular to chord $\Rightarrow$ bisects chord

diameter as hypotenuse $\Rightarrow$ right triangle in semicircle

Radians and Degrees

$$180^\circ = \pi \text{ radians}$$

$$\theta_{\text{rad}} = \theta_{\text{deg}} \cdot \frac{\pi}{180}$$

$$\theta_{\text{deg}} = \theta_{\text{rad}} \cdot \frac{180}{\pi}$$

Prisms and Cylinders

$$V_{\text{prism}} = Bh$$

$$V_{\text{cylinder}} = \pi r^2 h$$

$$SA_{\text{cylinder}} = 2\pi r^2 + 2\pi r h$$

Pyramids and Cones

$$V_{\text{pyramid}} = \frac{1}{3} Bh$$

$$V_{\text{cone}} = \frac{1}{3} \pi r^2 h$$

$$SA_{\text{cone}} = \pi r^2 + \pi r \ell$$

Spheres

$$V_{\text{sphere}} = \frac{4}{3} \pi r^3$$

$$SA_{\text{sphere}} = 4\pi r^2$$

Scale Factor

If linear scale factor is k:

$$\frac{\text{lengths}}{} = k$$

$$\frac{\text{areas}}{} = k^2$$

$$\frac{\text{volumes}}{} = k^3$$

Density

$$\rho = \frac{m}{V}$$

$$m = \rho V, \qquad V = \frac{m}{\rho}$$

Cheatsheet 11: Trigonometry and Right Triangle Tools

SOH-CAH-TOA

$$\sin\theta = \frac{\text{opposite}}{\text{hypotenuse}}$$

$$\cos\theta = \frac{\text{adjacent}}{\text{hypotenuse}}$$

$$\tan\theta = \frac{\text{opposite}}{\text{adjacent}}$$

Reciprocal Identities

$$\csc\theta = \frac{1}{\sin\theta}$$

$$\sec\theta = \frac{1}{\cos\theta}$$

$$\cot\theta = \frac{1}{\tan\theta} = \frac{\cos\theta}{\sin\theta}$$

Pythagorean Identity

$$\sin^2\theta + \cos^2\theta = 1$$

$$1 + \tan^2\theta = \sec^2\theta$$

$$1 + \cot^2\theta = \csc^2\theta$$

Tangent Ratio

$$\tan\theta = \frac{\sin\theta}{\cos\theta}$$

Complementary Angles

$$\sin(90^\circ - \theta) = \cos\theta$$

$$\cos(90^\circ - \theta) = \sin\theta$$

Exact Values

θ	30°	45°	60°
$\sin\theta$	$\frac{1}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$
$\cos\theta$	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$
$\tan\theta$	$\frac{\sqrt{3}}{3}$	1	$\sqrt{3}$

Law of Sines

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

Law of Cosines

$$c^2 = a^2 + b^2 - 2ab\cos C$$

$$a^2 = b^2 + c^2 - 2bc\cos A$$

Area with Trigonometry

$$A = \frac{1}{2}ab\sin C$$

where C is the included angle between sides a and b.

Slope and Angle

$$m = \tan\theta$$

$$\theta = \tan^{-1}(m)$$

Unit Circle Basics

$$(\cos\theta, \sin\theta)$$

$$\sin\theta = y, \quad \cos\theta = x, \quad \tan\theta = \frac{y}{x}$$

Common Radian Values

$$30^\circ = \frac{\pi}{6}, \quad 45^\circ = \frac{\pi}{4}, \quad 60^\circ = \frac{\pi}{3}, \quad 90^\circ = \frac{\pi}{2}$$

$$180^\circ = \pi, \quad 270^\circ = \frac{3\pi}{2}, \quad 360^\circ = 2\pi$$

Cheatsheet 12: SAT Word Problem Translation and Quick Traps

Translation Phrases

"is" $\Rightarrow =$, "of" $\Rightarrow \times$

"at least" $\Rightarrow \geq$, "at most" $\Rightarrow \leq$

"more than" $\Rightarrow +$, "less than" $\Rightarrow -$

Percent Word Problems

$$\text{“}p\%\text{ of }x\text{”} = \frac{p}{100}x$$

$$\text{“}p\%\text{ greater than }x\text{”} = x\left(1+\frac{p}{100}\right)$$

$$\text{“}p\%\text{ less than }x\text{”} = x\left(1-\frac{p}{100}\right)$$

Consecutive Integers

consecutive: $x, x+1, x+2$

consecutive even/odd: $x, x+2, x+4$

Age Problems

current age $= x$

age after t years $= x+t$

age t years ago $= x-t$

Rates and Units

$$\frac{\text{output}}{\text{input}} \cdot \text{new input} = \text{new output}$$

Always cancel units like fractions.

Inequality Trap

Multiply/divide by negative $\Rightarrow$ flip inequality sign

$$-3x \geq 12 \Rightarrow x \leq -4$$

Square Root Trap

$$\sqrt{x^2} = |x| \neq x \text{ always}$$

$$x^2 = 25 \Rightarrow x = \pm 5$$

$$\sqrt{x} = 5 \Rightarrow x = 25$$

Extraneous Solutions

Radical/rational equations may create false answers:

Check each solution in the original equation.

Function Trap

$$f(x+1) \neq f(x)+1 \text{ in general}$$

Substitute the full input into the function.

Average Trap

$$\frac{a+b}{2} \neq \frac{a}{b}$$

Average speed $\neq \frac{v_1+v_2}{2}$ unless times are equal.

Graph Trap

x-intercept uses $y = 0$

y-intercept uses $x = 0$

slope is $\frac{\text{vertical change}}{\text{horizontal change}}$

Calculator Check

Estimate first.

Reject impossible answers:

negative length, $P > 1$, percent out of context

Backsolving

When answer choices are numbers:

plug in middle choice $\Rightarrow$ adjust up/down

Plugging In Numbers

For variables in answer choices:

choose easy values such as $0, 1, 2, 10$

Avoid values that break denominators:

denominator $\neq 0$

Common SAT Setup Pattern

Define variable $\rightarrow x =$ unknown

Write equation $\rightarrow$ use phrase-by-phrase translation

Solve $\rightarrow$ simplify carefully

Answer requested $\rightarrow$ check whether they ask for $x, 2x, x+5$

One-Page Ultra Quick Formula Review

Algebra Core

$$ax + b = c \Rightarrow x = \frac{c-b}{a}, \quad m = \frac{y_2 - y_1}{x_2 - x_1}, \quad y = mx + b$$

$$|x - a| = b \Rightarrow x = a \pm b$$

$$\frac{a}{b} = \frac{c}{d} \Rightarrow ad = bc$$

Quadratics

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = -\frac{b}{2a}, \quad y = a(x-h)^2 + k$$

$$a^2 - b^2 = (a-b)(a+b)$$

Exponents

$$a^m a^n = a^{m+n}, \quad \frac{a^m}{a^n} = a^{m-n}, \quad (a^m)^n = a^{mn}$$

$$a^{-n} = \frac{1}{a^n}, \quad a^{m/n} = \sqrt[n]{a^m}$$

Functions

$$(f \circ g)(x) = f(g(x))$$

domain: denominator $\neq 0$

even root radicand ≥ 0

Data

$$\bar{x} = \frac{\sum x_i}{n}, \quad \text{IQR} = Q_3 - Q_1, \quad \text{residual} = y - \hat{y}$$

Probability

$$P(A^c) = 1 - P(A)$$

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$P(A \mid B) = \frac{P(A \cap B)}{P(B)}$$

Geometry

$$A_{\triangle} = \frac{1}{2}bh, \quad A_{\circ} = \pi r^2, \quad C = 2\pi r$$

$$a^2 + b^2 = c^2, \quad d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$M = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right)$$

Solids and Trig

$$V_{\text{cylinder}} = \pi r^2 h, \quad V_{\text{cone}} = \frac{1}{3}\pi r^2 h, \quad V_{\text{sphere}} = \frac{4}{3}\pi r^3$$

$$\sin\theta = \frac{opp}{hyp}, \quad \cos\theta = \frac{adj}{hyp}, \quad \tan\theta = \frac{opp}{adj}$$